MATHEMATICS

(Maximum Marks: 100)

(Time allowed: Three hours)

(*Candidates are allowed additional 15 minutes for **only** reading the paper.*

They must NOT start writing during this time.)

The Question Paper consists of three sections A, B and C.

*Candidates are required to attempt all questions from **Section A** and all questions
EITHER from **Section B OR Section C***

Section A: *Internal choice has been provided in three questions of four marks each and two
questions of six marks each.*

Section B: *Internal choice has been provided in two questions of four marks each.*

Section C: *Internal choice has been provided in two questions of four marks each.*

All working, including rough work, should be done on the same sheet as, and adjacent to

the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A (80 Marks)

Question 1 [10×2]

(i) If A = {1, 2, 3, 4, 5, 6,} B = {2, 4, 5, 6, 8, 9, 10}, find A Δ B.

(ii) Let A = {2, 4, 6, 8} B = {1, 2, 3, 4) and

R = {(a, b) : a ϵ A, b ϵ B, a is divisible by b}. Write Relation R in set builder
form.

(iii) Prove that:

$$\frac{Cos\ 2A}{1+Sin2A} = Tan\left(\frac{\pi}{4} - A\right)$$

(iv) In a Δ ABC, prove that $\frac{Sin\ A}{Sin(A+B)} = \frac{a}{c}$

(v) If $\frac{2+3i}{3-4i} = a + ib,$ find the values of a and b.

(vi) If α and β are the roots of the equation $Px^2 + qx + 1 = 0$, find
$\alpha^2\beta + \beta^2\alpha$.

(vii)   In how many ways can 12 books be arranged on a shelf if:

(a)   4 particular books must always be together.

(b)   2 particular books must occupy the first position and the last position.

(viii)   Find the derivative of : $\dfrac{x^4+3x^3+4x^2+2}{x^3}$

(ix)   Evaluate: $\displaystyle \lim_{x \to 0} \dfrac{(1-x)^n-1}{x}$

(x)   An urn contains 60 blue pens and 40 red pens. Half of the pens of each one is defective. If one pen is chosen at random, what is the probability that it is a defective or a red pen?

Question 2   [4]

Find the domain and range of : $2 - |x - 4|$

Question 3   [4]

(a)   Solve: $sin7x + sin\,4x + sinx = 0 \ and \ 0 < x < {}^{\pi}/_2$

OR

(b)   Prove that $\dfrac{\cos A + \cos 3A + \cos 5A + \cos 7A}{\sin A + \sin 3A + \sin 5A + \sin 7A} = \cot 4A$

Question 4   [4]

Using Mathematical induction, prove that $10^n + 3 \cdot 4^{n+2} + 5$ is divisible by 9 for an $n \epsilon N$.

Question 5   [4]

If $z = x + iy$ and $|2z - 1| = |z + 2i|$, find the locus of z and represent it in the argand diagram.

Question 6 [4]

(a)   A Committee of 6 members has to be formed from 8 boys and 5 girls. In how many ways can this be done if the Committee consists of :

(i)   Exactly 3 girls

(ii)   At least 3 girls

OR

(b)   How many different words can be formed of the letter of word "GRANDMOTHER", so that:

(i)   The word starts with G and end with R.

(ii)   The letters A, N, D are always together.

(iii) All vowels never come together.

Question 7 [4]

Find the term independent of x in the expression of :

$$\left(\frac{\sqrt{x}}{\sqrt{3}} + \frac{\sqrt{3}}{2x^2} \right)^{15}$$

Question 8 [4]

Find the equation of acute angled bisector of lines:

$3x - 4y + 7 = 0$ and $12x - 5y - 8 = 0$

Question 9 [4]

(a)   Find the equation of the tangent to the circle

$x^2 + y^2 - 2x - 2y - 23 = 0$ and parallel to line $2x + y + 3 = 0$

OR

(b)   Find the equation of the circle which passes through the points (2, 3), (4, 5) and the centre lies on the line $y - 4x + 3 = 0$.

Question 10 [4]

Differentiate the function Sin (2x – 3) by First Principle of differentiation.

Question 11 [6]

In a Δ ABC, $\dfrac{b^2-c^2}{b^2+c^2} = \dfrac{\sin(B-C)}{\sin(B+C)}$, prove that it is either a right angled or isosecles Δ.

Question 12 [6]

(a)   If 'x' be real, find the maximum and minimum value of: $y = \dfrac{x+2}{2x^2+3x+6}$

OR

(b)   If α, β be the roots $x^2 + lx + m = 0$, then form an equation whose roots are:

$$(\alpha + \beta)^2 \text{ and } (\alpha - \beta)^2$$

Question 13 [6]

(a)   The sum of three consecutive numbers of a G.P is 56. If we substract 1, 7 and 21 from these numbers in the order, the resulting numbers form an A.P., find the numbers.

OR

(b)   Find the sum of the series to n terms:

$$\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots\ldots\ldots\ldots \text{ n terms.}$$

Question 14 [6]

Find the mean, standard derivation for the following data:

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	2	3	5	10	3	5	2

SECTION B (20 Marks)

Question 15 [3×2]

(a)   Find the co-ordinates of a point on the parabola $y^2=8x$, whose focal dististance is 4.

(b)   Prove that:

$$\sim(P\Rightarrow q) = P^\wedge(\sim q)$$

(c)   Write Converse and inverse of the given conditional statement:

If a number n is even, then n^2 is even.

Question 16 **[4]**

(a) Find the equation of ellipse whose focus (1, 2), directrix $3x + 4y - 5 = 0$ and eccentricity is $\frac{1}{2}$

OR

(b) Find the centre, focus, eccentricity and latus rectrum of the hyperbola
$16x^2 - 9y^2 = 144$.

Question 17 **[4]**

(a) In what ratio the point P(–2, y, z) divides the line joining the points A(2, 4, 3) and B(–4, 5, –6). Also, find the coordinates of point P.

OR

(b) If the origin is the centroid of the triangle with vertices (–4, 2, 6) (2a, 3b, 2c) and (8, 14, –10) find the values of a, b and c.

Question 18 **[6]**

Find the equation of Parabola whose directrix is $2x - 3y + 4 = 0$ and vertex at (5 – 4)

SECTION C (20 Marks)

Question 19

(a) The mean weight of 150 students in a certain class is 60 kg. The mean weight of boys is 70 kg and that of girls in the class is 55 kg. Find the number of boys and girls in the class. **[2]**

(b) Compute D_3 and D_7 for the following distribution: **[4]**

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
No. of students	3	10	17	7	6	4	2	1

OR

Calcuate the mode from the following data:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of students	5	15	30	8	2

5

Question 20

(a) In a sample of 'n' observations given that $\sum d^2 = 55$ and rank correlation $r = \frac{2}{3}$, **[2]**
then find the value of 'n'.

(b) Find the correlation coefficient r(x, y) if: **[4]**

   $n = 10, \sum x = 60, \sum y = 60, \sum x^2 = 400, \sum y^2 = 580, \sum xy = 305,$

OR

Ten students got the following percentages of marks in Mathematics and Physics:

Mathematics	56	64	75	85	85	87	91	95	97	98
Physics	89	90	86	74	78	66	56	74	86	90

Find the Spearman's rank correlation coefficient for the above data.

Question 21 **[4]**

Calculate the index number for the year 1990 with respect to 1980 as base from the following data using weighted average of price relatives:

Commodity	Weights	Year 1990	Year 1980
A	22	320	200
B	48	120	100
C	17	20	28
D	13	60	40

Question 22 **[4]**

The number of letters, in hundreds, posted in a certain city on each day for a week is given below:

Mon.	Tue.	Wed.	Thur.	Fri.	Sat.	Sun
35	70	36	59	62	60	71

Calculate 3 day moving averages and represent these graphically.